Measuring the Earth’s Mass and Density

Using a simple pendulum, we will first determine the acceleration due to Earth’s gravity, and then calculate the Mass and Density of the Earth.

**MATERIALS**

1. Pendulum Kit (plastic line, weight and clip)
2. Meter Stick
3. Stopwatch (Refer to stopwatch use page.)
4. Graph Paper upon which to plot your data (page 4)
5. Calculator
6. Pencil (to record your data)

**BACKGROUND**

A pendulum swings with a **Period** determined only by its **Length**. The **Mass** of the attached weight has no effect! The relationship between a pendulum’s Length (**L**) and its Period (**T**) is given by:

\[
L = \left(\frac{g}{(2\pi)^2}\right) T^2
\]

(As long as the amplitude of the swing is small. See illustration below, right.)

Where **g** is the **Acceleration due to Gravity** at the Earth’s surface.

If we measure the Period (**T**) for different pendulum lengths (**L**), we can graph **L** vs. **T**^2, which will give a slope (**m**) of:

\[\text{m} = \left(\frac{g}{(2\pi)^2}\right) \text{ SLOPE}\]

Note the similarity of formula 1) to the general “**Slope**” formula,

\[y = mx + b\]

if \(y = L\); \(x = T^2\); and \(b = 0\)

Measuring the slope means we can solve for **g**:

\[g = \text{m} \cdot (2\pi)^2 \text{ SLOPE}\]
DETERMINING THE VALUE OF \( g \)

1. Make a series of observations where the Period \( T \) is measured for a given pendulum of varying Lengths \( L \); See in the table below, column 1.

2. For each given Length \( L \), measure the Period \( T \) by noting the total number of seconds it takes for 30 full swings of the pendulum. Enter your data in the 2nd column, “Observations 1”.

3. Repeat the above observations. Enter the Period data in the 3rd column, “Observations 2”.

4. Calculate the averages for the two sets of 30 swing Period measurements. Enter the results in column 4, “Avg. Period, 30 Swings”.

5. Calculate Each Period for One Swing by dividing the values in column 4 by 30. Enter the results in column 5, “Avg. Period, 1 Swing”.

   (Make sure you have done the calculations for each value of \( L \), \( 0.2 \) – \( 1.0 \) Meters).

6. In column 6, enter the Square of each of the \( T \) values from column 5.

<table>
<thead>
<tr>
<th>( L ) (Meters)</th>
<th>( T ) (Seconds) Observations 1</th>
<th>( T ) (Seconds) Observations 2</th>
<th>Avg. Period (T 30 Swings) (Seconds)</th>
<th>Avg. Period (T One Swing) (Seconds)</th>
<th>( T^2 ) (Seconds²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td></td>
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<td>0.6</td>
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<tr>
<td>0.8</td>
<td></td>
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</tr>
<tr>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. Plot* the Length \( L \) vs. the Period Squared \( T^2 \) for each set of data points on the graph below.

   Draw a “best fit” straight line using all the points, and then measure the slope \( m \).

   (Refer to the “Graphing Data” reference sheet) \[ \text{Slope} (m) = \text{______________________________} \]

   Notes: 1) On your graph, \( L \) should be the Y-axis (vertical), and \( T^2 \), the X-axis (horizontal).
   2) Remember to number and title the axes, and include the Units.
   3) Use the entire graph paper (not just a small corner.)
   * (If you prefer to plot your data in Excel Spreadsheet, you should use the “Linear Trend Line” option.)

CALCULATIONS

1. Calculate the Earth’s gravitational acceleration \( g \) from formula (2b):

   \[ g = \frac{m \ (2\pi)^2}{(\text{______}) \ (\text{______})^2} = \text{______________________________} \quad \text{[Units]} \]

   [Note: Don’t forget to include the units.]

2. From Newton’s Law of Universal Gravitation, the acceleration due to gravity at the Earth’s surface may be expressed as:

   \[ g = \frac{GM_e}{R_e^2} \]

   Where \( R_e \) is the Earth’s radius; and \( G \) is the Universal Gravitational Constant, you can now calculate the Mass of the Earth by solving for \( M_e \):

   \[ M_e = \frac{gR_e^2}{G} = \frac{(\text{______}) \ (\text{______})^2}{(\text{______})} = \text{______________________________} \quad \text{[Units]} \]
3. Calculate the Volume of the Earth, \( V_e \), using the volume formula in the Reference section below.

\[
V_e = \frac{4}{3} \pi R^3
\]

[Units]

4. The Density of the Earth (\( \rho_e \)) is its Mass (\( M_e \)) divided by its Volume (\( V_e \)):

\[
\rho_e = \frac{M_e}{V_e}
\]

Calculate the Density of the Earth. _______________________ 

[Units]

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**Reference Values and Formulae**

\( \pi (pi) = 3.14 \)

\( G \) (Universal Gravitational Constant) = 6.67 x 10\(^{-11}\) m\(^3\)/Kg s\(^2\).

\( R_e \) (Radius of the Earth) = 6.38 x 10\(^{3}\) Km = ______________________ m.

\( M_e \) (Mass of the Earth) = 5.97 x 10\(^{24}\) Kg.

\( V \) (The Volume of a sphere is determined by the following formula): \( V = \frac{4}{3} \pi R^3 \)

\( \rho \) (Density, is the Mass divided by the Volume: \( \rho = \frac{M}{V} \))

**Material Densities:**

- Water = 1000 kg/m\(^3\)
- Rock \( \approx \) 2200 - 3000 kg/m\(^3\)
- Metals \( \approx \) 7000 - 9,000 kg/m\(^3\)

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**QUESTIONS**

1. How do your results for \( M_e \), from this activity, compare with the accepted value above?

____________________________________________________________________________________

Express your error as “off by” ________% – i.e. \([\text{Accepted} \ M_e - \text{Your} \ M_e]/\text{Accepted} \ M_e \] \cdot 100

2. What do you think could be reasons for the error? ______________________________________

____________________________________________________________________________________

3. Had you performed this activity on the Mars, would you have obtained the same result? ________

Why? (Hint: Consider the relative gravitational accelerations, masses, and radii of the Earth and Moon.)

____________________________________________________________________________________

____________________________________________________________________________________

4. The density of Water = 1000 kg/m\(^3\); the average density range for Rock \( \approx \) 2200 - 3000 kg/m\(^3\); and the average density range for Metals (Nickel-Iron, etc) \( \approx \) 7000 - 9,000 kg/m\(^3\). Mark the Earth’s Density, \( \rho_e \), that you calculated on the Density Scale below.

**Density Scale**

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Based on the Density you indicated above, of what is Earth’s composition mostly comprised?

- Mostly Water
- Mostly Water & Some Rock
- Mostly Rock & Some Water
- Mostly Rock
- Mostly Rock & Some Metals
- Mostly Metals & Some Rock
- Mostly Metals

(circle one)